

Final Report

HF002: Applied Game Theory to Enhance Air Traffic Control Training

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ABSTRACT

As the demand of air transportation is growing year by year, conflict detection and resolution are becoming major concerns in air traffic management. To increase safety, efficiency, and flexibility in airspace traffic flow, innovative technological advances such as computerization is a must for the future of air traffic control (ATC) operations. In this study, we propose an algorithm based on game theory to resolve the problem of conflicts among multiple aircrafts moving in a shared airspace and provide a new set of maneuvers that deviate minimally from the original routes. The proposed model assists ATC to make decision efficiently in standard situations and provides insights for ATC training in decision making procedures in unusual circumstances in a timely manner.

KEYWORDS: Conflict detection and resolution, Air traffic control, Game theory.

1. INTRODUCTION AND MOTIVATION

Air traffic management is a challenging problem due to the growth of the number of aircraft and the interdependency among their decisions in the airspace (Dmochowski & Skorupski, 2017; Volpe Lovato, Hora Fontes, Embiruçu, & Kalid, 2018). It is estimated that the demand for air transportation in the United States will continue to grow by 5% annually (Jangam & Mumbai, 2013). Before 1950, the concept of air traffic control (ATC) was not as mature as today and flight controlling systems were only suitable for low air travel demands as its technology was limited to cockpit advising the pilots to avoid collisions based on collision avoidance system (Kraus, 2008). In fact, flight controllers with the help of radars were responsible for controlling a sector in the airspace, and their assistants circulated flight strips from a controller in a sector to others (Kraus, 2008). After 1960, a significant need for a form of ATC arose for conflict detection and resolution among aircrafts (Kraus, 2008; Schwab et al., 2015).

Traditionally, ATC was viewed as a centralized system to control the operation of aircraft and maintain smooth traffic in the airspace. Through this process, humans played an important role because of their ability to incorporate information (e.g., historical data and logic) to make effective decisions. Increasing airspace congestion and perhaps conflicting business plans across airlines, it was difficult for controllers to consider airline-driven criteria in making decisions because of various contradictory interests and complexities (Adacher & Meloni, 2005; Hansen, 2004; Jonker, Meyer, & Dignum, 2005). On the other hand, to make any predictions about the behavior of a system in general, a model is required to consider both the human and non-human aspects of that system. Therefore, innovative technological and computational advances to integrate information became a must for supporting ATCs to increase safety, efficiency, and flexibility in airspace traffic flow (Diao & Chen, 2018; Kraus, 2008). Therefore, the need has grown to develop new technologies, investing in infrastructures and technical training to assist ATC and reduce the controller's workload (Yildiz et al., 2014). In addition, many efforts have been made in the form of non-centralized control by relying on computerized models to ensure aircraft separation in the airspace dynamically and automatically (Adacher & Meloni, 2005; Jangam & Mumbai, 2013; Jonker et al., 2005). These recent arrangements have been developed under the concept of free flight (FF), which gives responsibility to pilots to select and change their own trajectory and flight characteristics independently under the constraints of traffic rules set by ATCs (Brady & Stolzer, 2017). Adopting FF allows airlines to prioritize criteria with respect to their business plan with more liberty.

As such, government-led improvements to technologies have been applied to improve ATC and pilot decision making, including instrumental landing systems, aircraft navigation systems, communication and data transfer systems, and management modernization programs (Baiada, 2017). These programs have been developed to consider FF concept as the vision to be achieved. All these efforts have been aggregated in a program entitled Next Generation Air Transportation System (NextGen) to provide safety for the FF concept, increase capacity for airspace, and enhance predictability and efficiency (Federal Aviation Administration, 2018). There are some technological requirements for NextGen that provide accurate and integrated information for planning and controlling the airspace. In addition, it ensures that all parties in the system have access to enough information such as aircrafts positions, flight characteristics, and weather situations, among others, for making decisions independently and effectively following their own interests (Wolfe, Jarvis, Enomoto, Sierhuis, & van Putten, 2009).

As mentioned, the ATC system is primarily designed to provide safety in airspace, maximize the flow of air traffic, and communicate successfully with pilots. In the ATC system, controllers are the human part of the system and major players to manage different situations (Yildiz et al., 2012). There are various technological advances to help them tackle the situation of conflicts. Although various approaches have been proposed in the literature to address the problem of conflict detection and resolution, there is still a need to address a larger set of issues including addressing unusual situations and handling multiple conflicts. Furthermore, because the airspace has a great potential for drawing an infinite number of routes from origin to destination, studies that consider only a limited number of routes sacrifice necessary details contributing to the efficiency of the system to provide simplicity for the model. However, such a lack of details would be counter to the goal of the FF concept, resulting in a substantial loss of efficiency in the system. However, theoretically, there exists the capability to define an infinite number of routes between an origin and destination pair. As such, optimization models assuming a limited number of routes from origin to destination lack the realism to make their solutions usable. In addition, due to the dynamic characteristic of airspace where each operator should be able to react to its instant situation, decision making techniques should deal with changing situations (e.g., (i) some parts of airspace may become unavailable and impose unpredicted traffic to other routes, (ii) since various aircraft move in and out the airspace, with the passage of time, the view of other aircraft will change and affect their decision procedures). However, an optimization approach individually cannot handle the system of multiple agents in the dynamic airspace, though agents are often willing to compromise to enhance their individual and group preferences instead of competing intensively over their interests.

Therefore, we propose an algorithm based on game theory to anticipate the possibility of conflicts among multiple aircraft moving in a shared airspace and provide a new set of routes to aid ATC in streamlining this process and predict the outcomes of complex system behavior specifically in unpredicted circumstances. As the decision making process among airplanes is mutually dependent, a game model is an appropriate method to analyze human behavior within air transportation. The game theory model is used to anticipate pilot behavior in en-route flying situations. This approach will enhance the cognitive abilities of the pilot and avoid potential collisions thereby improving flexibility in handling variations to normal conditions. The proposed model considers several constraints to simulate real-world air traffic challenges such as maintaining a minimum required distance among aircraft and improving efficiency. The primary focus of this study is to understand how pilots would resolve en-route spatial conflicts on their own such that ATCs can learn from their decision-making process. Such insights can be used to guide ATC decision for conflict resolution that account for the perspective of airlines, while maintaining FAA rules. This study generally improves decision making strategies in scenarios with humans involved. The efficiency of this model affects the efficiency of the air traffic controllers as they are the main components of the air traffic management. The main contributions of this model are to consider (i) non-centralized decision making procedures, (ii) ATC decision making in nominal situations and deviations from those nominal situations, (iii) infinite possible routes, (iv) dynamic aspects of the airspace, and (v) ATC as a facilitator providing coordination among aircraft.

The remainder of this study is organized as follows. In Section 2, a review of conflict detection and resolution methods for air traffic management is presented. The proposed algorithm based on game theory is discussed in Section 3. Illustrative examples and computational results are presented in Section 4 to show the application of the proposed model. Section 5 provides concluding remarks of this work.

2. RESEARCH BACKGROUND

In air traffic management, a conflict is generally referred to as an event wherein two or more aircraft lose their minimum separation. In the airspace, two cylinders are assumed to encircle each aircraft: the protected zone and the alert zone, such that the occurrence of overlap between protected zones results in conflict recognition or loss of separation among aircraft (Tomlin et al., 1997, 1998; Xu et al., 2016).

Due to the increase of airspace congestion, the need for advanced technologies is a growing concern to aid air traffic controllers in conflict detection and resolution for enhancing safety and traffic flow efficiency. The problem of conflict detection and resolution in air traffic management has been extensively addressed with different perspectives in the literature (Hong & Harker, 1992; Menon et al., 1999; Sastry et al., 1995; C. Tomlin et al., 1997, 1998). There are various methods proposed by the researchers to maintain the separation among airplanes (Bicchi & Pallottino, 2000; Bilimoria, 2000; Jangam & Mumbai, 2013; Pallottino et al., 2002; Tripathy et al., 2016; Wollkind et al., 2004; Xu et al., 2016). The key features and assumptions distinguishing these studies can be described as (i) centralized or non-centralized decision making procedures, (ii) different navigation assumptions depending on if an aircraft selects from a certain number of routes or if an infinite number of routes exist, and (iii) information awareness by all parties including aircraft and ATC.

Optimization and modeling approaches have been widely applied in this area of research. Kuchar and Yang (2000) provided a review of 68 research works about modeling methods for air traffic conflict detection and resolution. In this survey, models are categorized with respect to principal design factors. Note that suggested approaches have also been applied to other modes of transportation (e.g., land and marine navigation) since the nature of conflict prevention problems have similar characteristics regardless of mode (Zhang et al., 2010). Menon et al. (1999) resolved the problem of multiple aircraft conflicts with a trajectory optimization formulation. Since aircraft flying in the shared airspace follow contradictory preferences, the conflict issue was formulated as a multi-participant optimization problem such that each airplane involved in a possible conflict aims to maximize its interests while keeping allowable separation. Bilimoria (2000) proposed an optimization model based on the geometric characteristics of aircraft routes. In this study, the optimal solution of heading and speed is provided in the horizontal plane to resolve conflict issues among aircraft successively according to their priority. Pallottino et al. (2002) addressed the problem of conflict resolution with mixed integer optimization model, considering the shortest path problem to find conflict-free routes for multiple airplanes with the aim of minimizing the total flight time.

Optimization methods for maintaining separation among airplanes are not limited to mathematical modeling, as other techniques such as genetic algorithms, game theory, expert systems, and fuzzy control have been also applied to the problem of conflict avoidance in air traffic management

(Kuchar & Yang, 2000). For example, Hill et al. (2005) applied satisficing game theory to provide a decision strategy for pilots to cooperate in satisfying both individual and group preferences. Likewise, Bellomi et al. (2008) applied a similar approach to resolve conflict detection in airspace such that the preference of each individual, as well as group, are achieved. Xu et al. (2016) resolved the problem of conflict detection in air traffic management by a probabilistic prediction model with the application of a cooperative game model in which the suggested conflict-free maneuver provides the global benefit, including total flight time and fuel consumption for all players. The study conducted by Wollkind et al. (2004) proposed a conflict resolution technique based on cooperative multi-agent negotiation, where agents negotiate by the monotonic concession protocol to specify a conflict-free and acceptable maneuver in the situation of conflict detection. The flow scheduling problem in air traffic management can be considered as a conflict detection and resolution problem due to the features similar to both problem statements. For example, Pias et al. (2016) modeled the problem of aircraft scheduling with a mixed integer programming formulation with the aim of minimizing total delay time for the aircraft. Since the problem is involved for different airlines with various preferences, a cooperative game model was applied to solve the interaction among players. Likewise, Wang (2010) formulated this problem as a multi-player dynamic game since different airlines compete for their preferences by finding an optimal maneuver.

While cooperative decision making strategies are broadly applied to conflict issues in air traffic management (Ball et al., 2001), the research conducted by Tomlin et al. (1997) suggested a noncooperative conflict resolution model. In this study, it was assumed that the current situation of every aircraft is known for other airplanes involved in a potential conflict, but the plan of every other aircraft is not identified because of miscommunication problems or technological malfunctions. A zero-sum noncooperative approach based on dynamic game theory was applied to tackle this situation. Some other studies are summarized in Table 1 presenting different optimization methods to the problem of conflict resolution and airplane scheduling.

3. METHODOLOGY

Game theory was originally developed by Von Neumann and Morgenstern (1944) to analyze strategic decision making among a group of players. Typically, players represent human decision makers though the concept of a player can be extended to other decision makers including complex automated decision makers (Yildiz et al., 2014). The decisions of players are driven by a reward function, often referred to as a utility function. The best action of a player will depend on other players who are a part of the decision environment. “Game theory is the logical analysis of situations of conflict and cooperation” (Straffin, 1993). This, in detail, can be explained as a situation where there are at least two players, and each player has many possible strategies that determine the outcome of the game, where the outcome is determined by the value, or payoff, of the decision to the player. Game theory assumes rational game play as the outcome is equally dependent on all the players involved in the game. In a rational game, players should decide on whether to cooperate or not, which can require complicated decision making about the game (Zhang et al., 2010b).

Table 1: Summary of previous research work.

No.	Title	Author	Year	Method	Features, assumptions
1	A sequence model for air traffic flow management rerouting problem	Diao & Chen	2018	Integer optimization model	Rerouting, ground-holding delay, fuel consumption and flight cancellation
2	A fuzzy modeling approach to optimize control and decision making in conflict management in air traffic control	Volpe Lovato et al.	2018	Fuzzy model	Quantifying the longitudinal conflict levels between two aircraft, longitudinal acceleration of the aircraft
3	Scheduling models for optimal aircraft traffic control at busy airports: Tardiness, priorities, equity and violations considerations	Samà et al.	2017	Mixed integer linear programming formulations	The real-time optimization of take-off and landing operations
4	Predicting Pilot Behavior in Medium-Scale Scenarios Using Game Theory and Reinforcement Learning	Yildiz et al.	2014	Multi-stage game theory, agent-based simulation	Reinforced learning
5	Innovation in air traffic strategy using game theory	Jangam & Mumbai	2013	Multi-stage game theory, agent-based simulation	Focusing on FF concept, all parties are fully informed
6	Game theoretic modeling of pilot behavior during Mid-Air encounters	Lee & Wolper	2012	Semi network-form Games	Focusing on collision avoidance system
7	Using Game Theoretic Models to Predict Pilot Behavior in NextGen Merging and Landing Scenario	Yildiz et al.	2012	Semi network-form games	Aircrafts competing in approaching to the merging point
8	A Multi-Agent Simulation of Collaborative Air Traffic Flow Management	Wolfe et al.	2009	Multi-stage game theory, agent-based simulation	Collaborative air traffic flow management

In this study, we propose an algorithm based on game theory to resolve the problem of conflicts among multiple aircrafts moving in a shared airspace and provide a new set of maneuvers that deviate minimally from the original routes. The proposed algorithm includes three major stages: (i) dealing with the primary constraint of predicting the potential conflicts in the airspace where airplanes may violate the safety distances requirements, (ii) maximizing efficiency while maintaining safety measures by utilizing game theory, and (iii) resolving conflicts by rerouting aircraft with respect to the outcome of the second stage.

Figure 1 presents the general framework of the proposed algorithm. Independent conflict zones (CZs) in the airspace are identified where airplanes violate their minimum separation. In the next stage, a game model is constructed for each independent CZ in the list and a new set of aircraft

maneuvers are presented. New routes provide either corrected path or velocity change for each airplane with the aim of having minimum deviation from the original plan. As such, the options presented to each pilot deal with (i) a deviation of path and/or (ii) a deviation of velocity. The procedure is repeated until no CZ is found. In the following, assumptions and different stages of the algorithm are presented in detail.

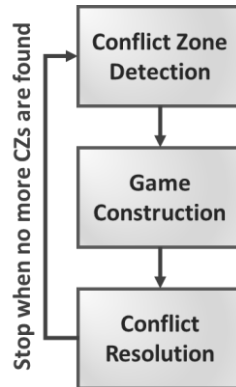


Figure 1: Algorithm framework.

3.1. Algorithm Assumptions

The proposed algorithm includes some assumptions as follows:

- The airspace is approximated with a two-dimensional (2-D) grid.
- Aircraft have the tendency to select the immediate available direct path towards their destination.
- Aircraft must keep a minimum safe distance from each other.
- There are pre-defined routes for the travel plan of each aircraft as an input to the algorithm.
- Pilots are expected to continue flying on their assigned trajectory as long as maintaining minimum separation from other aircraft.
- CZs can be avoided when aircraft deviate from their intended route or from their intended speed.
- Aircraft are assumed to be at en-route phase of the flight and keep flying at the same altitude throughout the scenario.

3.2. Conflict Zone (CZ) Detection

A conflict zone (CZ) is defined as the point in the airspace where the routes of aircraft violate the minimum safe distance. In this phase of the model, the aircraft flight plan (or specifically the flight route) is the input to the algorithm. The flight route consists of waypoints in the 2-D airspace that aircraft should pass from origin to destination. For example, Figure 2 depicts flight routes near Schiphol Airport in Amsterdam, The Netherlands, in which NIRSI, KIAS, NARIX, SOKSI, PAM, and ARTIP represent waypoints (Liu & Hwang, 2011a). A direct path from one waypoint to another is defined as route segment. To arrive at the destination, aircraft must pass a number of maneuvers following the given route segments.

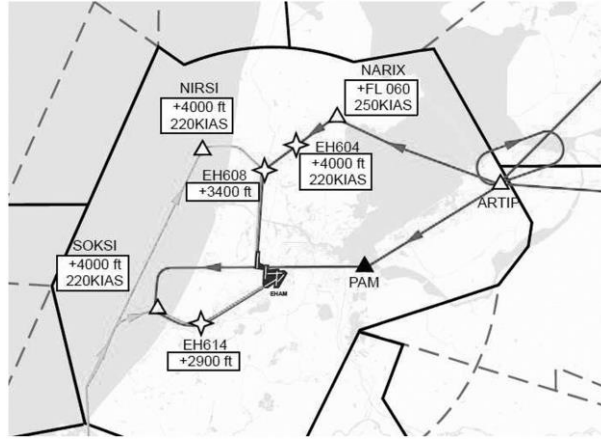


Figure 2: Flight routes close to Schiphol Airport in Amsterdam (Liu & Hwang, 2011b).

The route segments overlapping each other may result in conflict among aircraft. As such, to find the primary candidates for CZs, line cross points in direction of each two route segments are calculated as shown in Eqs. (1) and (2), where (x_c, y_c) refers to the coordinates of the line cross point, m_1 and m_2 represent the slopes of route segments 1 and 2, respectively, and (x_1, y_1) and (x_2, y_2) are the coordinates of the waypoints related to each route segment.

$$x_c = \frac{m_1 x_1 - m_2 x_2 - y_1 + y_2}{m_1 - m_2} \quad (1)$$

$$y_c = \frac{m_2 y_1 - m_1 y_2 + m_1 m_2 x_2 - m_1 m_2 x_1}{m_2 - m_1} \quad (2)$$

If (x_c, y_c) falls within the starting and ending points of route segments, it is considered the primary candidate for CZ.

Regardless of aircraft trajectory, aircraft velocity is another important factor that needs to be considered for the conflict detection in the airspace. Aircraft velocity determines if the airplanes are going to be at close proximity to the route segment cross point at the same time, such that a collision would be inevitable. To check the possibility of a collision at the route segment cross point, the minimum distance that airplanes could have, based on their velocity, is calculated. If the minimum distance is less than the minimum safe distance among airplanes, this cross point is considered as the CZ.

According to classical mechanics, velocity is the rate of change in the position with respect to the time. In this problem, aircraft position, trajectory and velocity are given. Eq. (3) calculates the required time for travel, where Δt , Δx , and v stand for time variation, position variation, and velocity, respectively.

$$\Delta t = \frac{\Delta x}{v} \quad (3)$$

Furthermore, the distance, d , between two objects (points) is calculated with the Euclidian metric represented in Eq. (4) in which x and y refer to the longitude and latitude positions of aircraft i and j , respectively by considering a 2-D airspace.

$$d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (4)$$

Note that the aircraft position in the airspace is changing constantly, therefore by rewriting aircraft position with respect to time, t , Eqs. (5) and (6) are obtained, where β_i , x_{i0} , and y_{i0} refer to the heading, primary longitude coordinate, and primary latitude coordinate for aircraft i by considering a 2-D airspace.

$$x_{it} = v_i \cos(\beta_i) t + x_{i0} \quad (5)$$

$$y_{it} = v_i \sin(\beta_i) t + y_{i0} \quad (6)$$

The insertion of Eqs. (5) and (6) into Eq. (4) results in the formula for calculating the Euclidian distance between two aircraft with respect to time. To calculate the time in which airplanes have their minimum distance from each other and the value of this distance, we take a derivative of the Euclidian distance formula with respect to time, t , and set it equal to zero. Eq. (7) calculates the time, t_{\min} , that two aircraft have the minimum distance from each other.

$$t_{\min} = \frac{2 [v_i \cos(\beta_i) - v_j \cos(\beta_j)] \times [x_{i0} - x_{j0}] + 2 [v_i \sin(\beta_i) - v_j \sin(\beta_j)] \times [y_{i0} - y_{j0}]}{2 [v_i \cos(\beta_i) - v_j \cos(\beta_j)]^2 + 2 [v_i \sin(\beta_i) - v_j \sin(\beta_j)]^2} \quad (7)$$

Eq. (8) calculates the distance between aircraft at t_{\min} . The calculated distance at t_{\min} is compared with the minimum safe distance set by the ATC to check the possibility of a loss of separation. If the safety distance is violated, the associated point is considered as a CZ of the problem.

$$d = \sqrt{(v_i \cos(\beta_i) t_{\min} + x_{0i} - v_j \cos(\beta_j) t_{\min} - x_{0j})^2 + (v_i \sin(\beta_i) t_{\min} + y_{0i} - v_j \sin(\beta_j) t_{\min} - y_{0j})^2} \quad (8)$$

Mentioned previously, the occurrence of a number of CZs are subject to the course of action that is taken earlier in previous CZs representing as dependent CZs. CZs that do not have the predecessor relationship are listed as independent CZs. Two main factors are involved in categorizing CZs into dependent and independent categories. The most important factor is the t_{\min} of aircraft for each primary CZ. Earlier incidents are more likely to be a proper candidate for independent CZ. The second factor refers to incident lineage. Incident lineage describes the case when any of the aircraft involved in a specific CZ has been engaged in at least one CZ occurred earlier. A schematic illustration of CZ detection in this algorithm is shown in Figure 3.

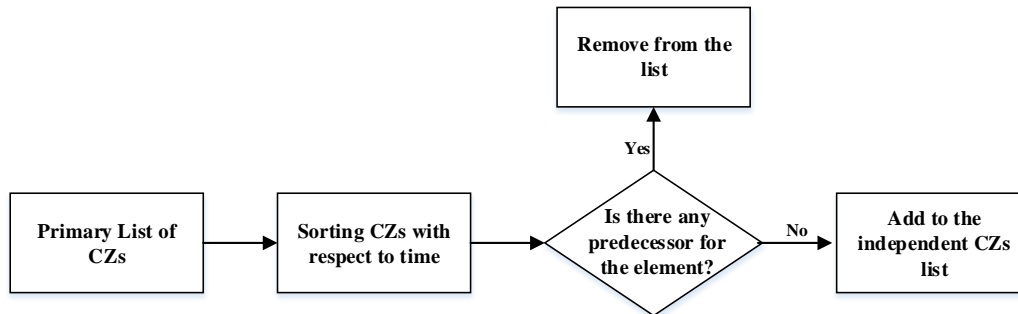


Figure 3: Conflict Zones (CZs) detection structure

3.3. Game Construction

In this stage of the algorithm, the game model is constructed for every member of the independent CZ list. Pilots are the players of the game. In game theory, players are not always able to maximize their reward function (Yildiz et al., 2014). Likewise, pilots in this study need to compromise to avoid collision. Meanwhile, an ATC enters the game to provide coordination among aircraft, to enhance both their individual and group preferences as well as to achieve safety measures. The strategies and the payoff functions are other principal elements of the game model that are discussed subsequently.

3.3.1. Strategies

The strategies in the game model refers to the group of all options existing for all players. In this study, there are two types of strategies coming in a certain combination with each other. The first type is related to the aircraft trajectory, and the second is associated with its velocity. For trajectory, it is assumed that each aircraft can move either straight or diagonally defined as (i) keeping the path, (ii) turning right with a certain degree, and (iii) turning left with a certain degree. For the speed, each aircraft can (i) keep the current velocity, (ii) decrease the velocity, and (iii) increase the velocity. The game constructed for this study accounts for 25 strategies consisting of five levels for the path change and five levels for the speed change. Table 2 details the 25 individual strategies that each player (pilot) could follow in our model. Therefore, if m be assumed as the number of pilots, then 25^m is the total number of possible different strategy combinations for the game. Note that the strategies can be easily extended to account for more levels regarding both direction and velocity, to give a better decision resolution, though the number of individual strategies and the size of the problem increases naturally.

3.3.2. Payoff Function

The payoff function is the third element of the game model. In the case of this study, appropriate payoff function for each player can be determined in various ways. There are many criteria affecting the nature of the payoff function for each player. The payoff function can be constructed based on various competitive strategies pursued by the airline and aircraft physical characteristics. Specifically, the payoff function in this industry is a product of two main criteria including distance traveled by the aircraft and their velocity (Krozel & Peters, 1997). These two criteria directly influence aircraft fuel consumption and flight duration time, both leading to flight operating costs (Liu & Hwang, 2011b). On the other hand, safety has the utmost importance for airlines. Collisions

could have a catastrophic impact on the community, as well as not only the airlines involved, but the entire airline industry and the FAA, and as such cannot be tolerated under any circumstances. Consequently, the payoff function relatively resonates with the variation in aircraft travel length, velocity, and delays. In addition, the aircraft collision imposes a virtual infinite cost to the payoff function.

Table 2: Game strategies for each player.

Strategies (s_i)	(Heading, Velocity)	Description
s_1	$(-20^\circ, -40\%)$	Turn 20° to right while decreasing 40% of velocity
s_2	$(-20^\circ, -20\%)$	Turn 20° to right while decreasing 20% of velocity
s_3	$(-20^\circ, 0\%)$	Turn 20° to right while keeping the velocity
s_4	$(-20^\circ, +20\%)$	Turn 20° to right while increasing 20% of velocity
s_5	$(-20^\circ, +40\%)$	Turn 20° to right while increasing 40% of velocity
s_6	$(-10^\circ, -40\%)$	Turn 10° to right while decreasing 40% of velocity
s_7	$(-10^\circ, -20\%)$	Turn 10° to right while decreasing 20% of velocity
s_8	$(-10^\circ, 0\%)$	Turn 10° to right while keeping the velocity
s_9	$(-10^\circ, +20\%)$	Turn 10° to right while increasing 20% of velocity
s_{10}	$(-10^\circ, +40\%)$	Turn 10° to right while increasing 40% of velocity
s_{11}	$(0^\circ, -40\%)$	Keep path while decreasing 40% of velocity
s_{12}	$(0^\circ, -20\%)$	Keep path while decreasing 20% of velocity
s_{13}	$(0^\circ, 0\%)$	Keep path and keep velocity
s_{14}	$(0^\circ, +20\%)$	Keep path while increasing 20% of velocity
s_{15}	$(0^\circ, +40\%)$	Keep path while increasing 40% of velocity
s_{16}	$(+10^\circ, -40\%)$	Turn 10° to left while decreasing 40% of velocity
s_{17}	$(+10^\circ, -20\%)$	Turn 10° to left while decreasing 20% of velocity
s_{18}	$(+10^\circ, 0\%)$	Turn 10° to left while keeping velocity
s_{19}	$(+10^\circ, +20\%)$	Turn 10° to left while increasing 20% of velocity
s_{20}	$(+10^\circ, +40\%)$	Turn 10° to left while increasing 40% of velocity
s_{21}	$(+20^\circ, -40\%)$	Turn 20° to left while decreasing 40% of velocity
s_{22}	$(+20^\circ, -20\%)$	Turn 20° to left while decreasing 20% of velocity
s_{23}	$(+20^\circ, 0\%)$	Turn 20° to left while keeping of velocity
s_{24}	$(+20^\circ, +20\%)$	Turn 20° to left while increasing 20% of velocity
s_{25}	$(+20^\circ, +40\%)$	Turn 20° to left while increasing 40% of velocity

In this study, the main idea behind the cost function is related to the amount of extra work that an airplane needs to perform to change its own flight characteristics. In general, the amount of thrust force generated by an aircraft is equal to airplane drag force acting in opposite direction of the thrust force if airplane is to maintain a constant speed. On the other hand, the drag force, D , has a direct relationship with the square of the airplane velocity, V , as mentioned in Eq. (9) according to the NASA website (Benson, 2014). Other parameters in this equation are related to the airplane aerodynamic characteristics, C_d , air density, r , and airplane wing area, A . Note that these parameters are considered constant in this study.

$$D = C_d \frac{rV^2}{2} A \quad (9)$$

The aircraft thrust force, F , is a relative variable of the square of velocity ($F \propto V^2$). According to the work formula in physics, the amount of work for an aircraft is relative to the amount of velocity multiplied by the movement distance, X , as represented in Eq. (10).

$$W = F.X \propto V^2.X \quad (10)$$

Therefore, the cost function is developed based on the amount of work performed by considering both velocity and path changes in airplane as shown in Eq. (11).

$$\text{Cost} = (V_{ab}X_{ab} + V_{bc}X_{bc}) - V_{ac}X_{ac} + I \quad (11)$$

To avoid collision, it is assumed that airplanes turn to a side at point a , which results in deviating from their original paths. After spending some time cruising in their new path, at point b , they turn back toward point c which was their original destination. As such, V_{ab} and X_{ab} are the aircraft velocity and travel length, respectively, in the deviating path from point a to point b , V_{bc} and X_{bc} represent the aircraft velocity and travel length, respectively, in correcting path from point b to point c , and V_{ac} and X_{ac} refer to the original velocity and the original travel length of the aircraft from point a to point c in the primary path. In addition, I in Eq. (11) is the index representing the possibility of an aircraft collision through its trajectory. In the situation of a collision, the value of I is set to a large number (representing infinity), otherwise it is 0.

An important question arises regarding where the airplanes start resolving the conflicts by adjusting their flight characteristics. In this study, the route segments involved in the CZs are considered to be the scope of the problem. Note that aircraft do not arrive simultaneously at the route segments involved in a specific conflict. Therefore, the position of the aircraft that arrives first should be adjusted on its route segment by considering the arrival time of the second aircraft at its route segment. Therefore, the arrival time of the latter aircraft is considered to be the start time for the conflict resolution algorithm.

After adjusting the flight plan of the airplane for resolving the conflict, the turning point should be determined (i.e., where the airplane needs to turn back the course towards its destination waypoint). The turning point position is required for the calculation of the travel length, and subsequently the payoff function. Although the turning point should be determined in such a way

to minimize the travel length, the safety distance among aircraft should not be compromised. Otherwise, a collision would occur. To avoid these situations, we assume the cost of a collision to be infinite. Finding an optimum value for the turning point is analytically complex since the Euclidean metric in the calculation of the cost function is nonlinear, and the collision index, I , discretizes the cost function. So, the ant colony algorithm is applied as an effective metaheuristic approach to optimize the position of the turning points. After this stage, the payoff function for the decision strategy made by every aircraft is calculated.

3.4. Conflict Resolution

In this part of the algorithm, the Nash equilibrium solution for each independent CZ is obtained. In game theory, the Nash equilibrium is a solution of a game including two or more players in which each player's strategy is optimal when considering the decisions of other players. In this situation, every player wins since individuals take the best outcomes which can be attained considering other players movements (Orsini et al., 2004; Straffin, 1993). For each independent CZ, multiple Nash equilibria are possibly found, therefore the ATC can select the best Nash equilibrium based on their priorities and decision criteria. Criteria can be (i) the total cost of a deviation from the original routes for both airplanes, (ii) the size of the aircraft and the number of passengers aboard, and (iii) the amount of cumulative delay, among others. So, the ATC coordinates the aircraft to maintain the interests and airspace priorities of both airlines. For this project, in the case of several Nash equilibria, the solution with the minimum summation of the payoff function for both aircraft is selected.

Afterward, the results of corrected paths and velocities change are substituted into the airplanes flight characteristic input data and the next iteration of the algorithm commences until no independent CZs are found. We reach to a solution for a static situation of the airspace. The algorithm restarts again at each time interval set by the decision maker so the new aircraft entering and exiting the airspace are considered in the procedure of conflict detection and resolution.

4. ILLUSTRATIVE EXAMPLE

To illustrate the application of the proposed algorithm, we study a segment of US airspace in the state of Florida considering 24 airplanes flying with different velocities. Figure 4 illustrates the segment of airspace studied in this work as well as the corrected flight paths for each airplane involved in a potential conflict. This example was generated based on an air traffic application created by Flight APP for Android. For this example, five miles is considered to be the minimum safety distance among airplanes. For the experiment, the game algorithm is implemented in Python 3.6.5 on a 64-bit Intel® Core™ m3-6Y30 CPU @ 0.90GHZ laptop computer.

The game for this example accounts for 25 strategies, consisting of five levels for the path change and five levels for the speed change. Since there are two aircraft involved in every CZ, the number of options in a strategy profile are 25^2 for each game.

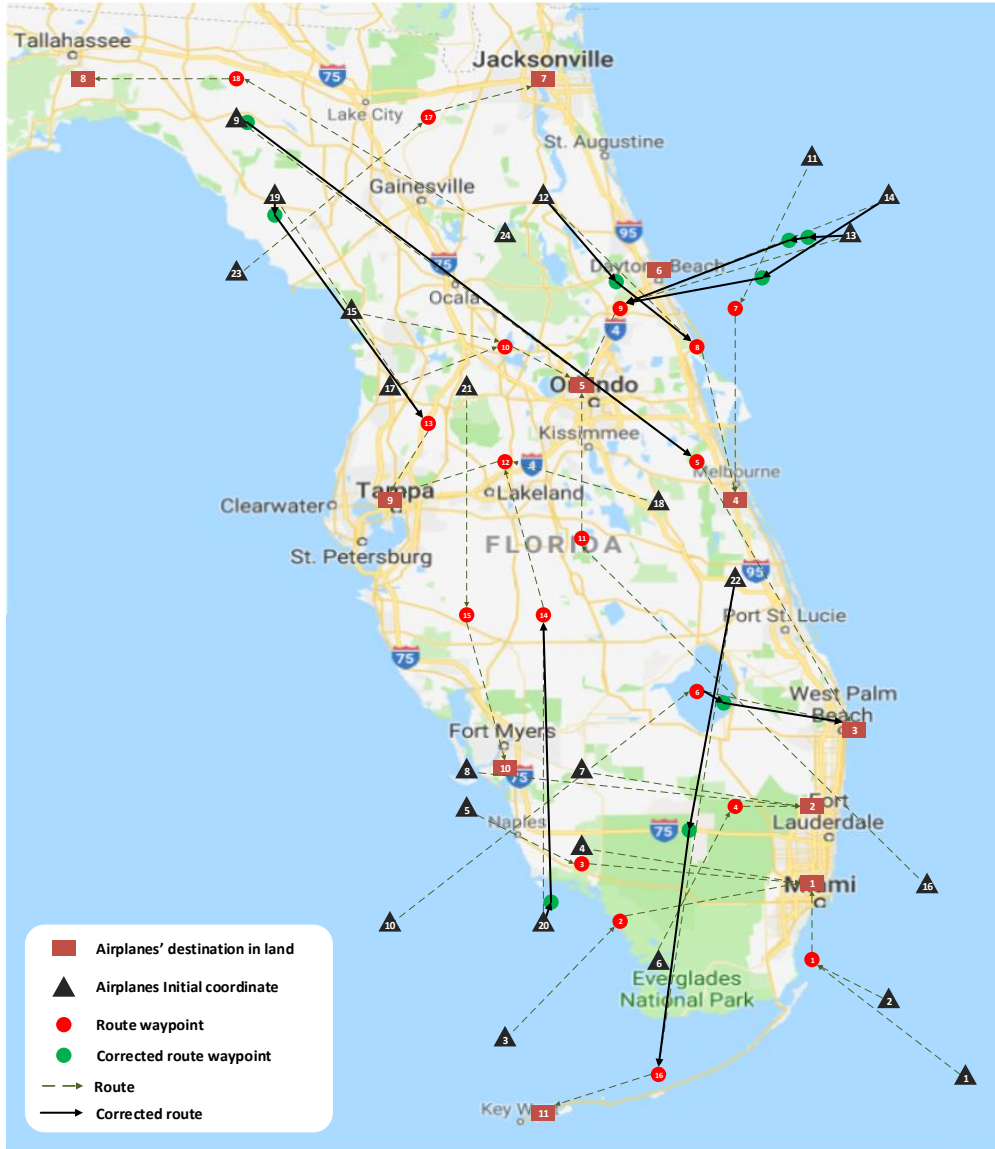


Figure 4: Original flight routes vs corrected flight routes in a segment of airspace over Florida.

Table 3 shows 10 potential CZs among aircraft and effective decision strategies made to resolve the conflicts in this example. Note that the total cost imposed on all airplanes for changing their flight characteristics is considered to be an indicator to represent the variation expense from the original flight plan. For example, from Table 3, it can be seen that there is a potential conflict between aircraft A_{19} and A_{23} , associated with their route segments number 0. To resolve this conflict with minimal deviations from the original routes, the model suggests that airplane A_{19} selects strategy s_3 , defined as turning 20° to right while keeping the velocity, and airplane A_{23} selects strategy s_{13} , defined as keeping both path and velocity. This conflict resolution strategy imposes $70.33 \text{ mi}^2/\text{hr}$ “cost” in total to both airplanes, where the term “cost” broadly refers to a penalty amount (in the basic sense, a quantity whose minimum is desired and where larger values are disincentivized). To resolve all CZs in this experiment, the suggested strategies impose $2083.16 \text{ mi}^2/\text{hr}$ to the system as the total variation cost.

Table 3: The critical zones (CZs) and conflict resolution strategies in a segment of Florida state airspace.

CZ	Aircraft IDs	Route segment	Decisions by aircraft	Cost for each aircraft	Total cost
1	[A ₁₉ , A ₂₃]	[0, 0]	(-20°, 0%), (0°, 0%)	[70.33, 0.0]	70.33
2	[A ₅ , A ₂₀]	[0, 0]	(0°, 0%), (-10°, 0%)	[0.0, 40.07]	40.07
3	[A ₁₁ , A ₁₄]	[0, 0]	(0°, 0%), (+10°, 0%)	[0.0, 775.97]	775.97
4	[A ₁₀ , A ₁₆]	[1, 0]	(0°, 0%), (-10°, 0%)	[0.0, 798.15]	798.15
5	[A ₃ , A ₂₂]	[1, 0]	(0°, 0%), (-10°, 0%)	[0.0, 54.65]	54.65
6	[A ₉ , A ₂₃]	[0, 0]	(+10°, 0%)(0°, 0%)	[39.18, 0.0]	39.18
7	[A ₁₃ , A ₁₄]	[0, 0]	(-10°, 0%), (0°, 0%)	[86.36, 0.0]	86.36
8	[A ₁₁ , A ₁₃]	[0, 1]	(-10°, -40%), (0°, 0%)	[0.0, 687.66]	687.66
9	[A ₁₃ , A ₁₄]	[1, 0]	(-20°, 0%), (0°, 0%)	[85.92, 0.0]	85.92
10	[A ₁₂ , A ₁₃]	[0, 3]	(-10°, 0%), (0°, 0%)	[113.92, 0.0]	113.92

There are several special incidents that can cause a deviation from normal operations in the airspace (e.g., weather conditions, technical issues, other security related issues). A lack of available airports, the occurrence of emergency landings, and increased airspace congestion are the main outcomes of such urgent circumstances. In the following sections, two major scenarios are discussed in detail.

4.1 High Airspace Congestion

In this scenario, the number of airplanes flying in the airspace increases by around 40% with respect to the primary illustrative example (from 24 to 34 airplanes) which results in high airspace congestion. In this situation, the number of potential CZs increases as well from 10 to 14. Consequently, the total cost imposed to the system to resolve all CZs increases by around 20%, from 2083.16 mi²/hr to 2463.32 mi²/hr. Note that, in this scenario, 5 miles is again considered as the minimum separation distance. Table 4 shows the potential CZs between airplanes in this situation, as well as the decisions made to resolve the conflicts.

4.2 Closing the Airspace

In general, clearing the airspace from all flying objects is an important, though rare, situation. For example, following the terrorist attack on September 11, 2001, the entire airspace of the United States was closed for at least two days except for military, police, and medical purposes by order of the FAA (Comfort & Kapucu, 2006).

To represent this scenario, it is assumed that the airspace is closed to any aircraft to enter and exit. As such, aircraft flying in the airspace should be landed at the nearest airport. The minimum separation for the aircraft is assumed 5 miles in this scenario. Table 5 shows potential conflicts among aircraft in this situation, as well as decisions made to resolve the conflicts.

Table 4: The critical zones and conflict resolution strategies under high airspace congestion scenario.

CZ	Aircraft IDs	Route segment	Decisions by aircraft	Cost for each aircraft	Total cost
1	[A ₁₉ , A ₂₃]	[0, 0]	(-20°, 0%), (0°, 0%)	[70.55, 0.0]	70.55
2	[A ₅ , A ₂₀]	[0, 0]	(-10°, 0%), (0°, 0%)	[0.0, 39.26]	39.26
3	[A ₁₁ , A ₁₄]	[0, 0]	(0°, 0%), (+10°, 0%)	[0.0, 775.97]	775.97
4	[A ₁₈ , A ₂₉]	[0, 1]	(0°, 0%), (-10°, 0%)	[0.0, 92.41]	92.41
5	[A ₂₁ , A ₂₇]	[0, 0]	(0°, 0%), (+10°, 0%)	[0.0, 282.37]	282.37
6	[A ₁₀ , A ₁₆]	[1, 0]	(-10°, 0%), (0°, 0%)	[128.37, 0.0]	128.37
7	[A ₃ , A ₂₂]	[1, 0]	(0°, 0%), (-10°, 0%)	[0.0, 54.66]	54.66
8	[A ₉ , A ₂₃]	[0, 0]	(+10°, 0%), (0°, 0%)	[36.25, 0.0]	36.25
9	[A ₁₃ , A ₁₄]	[0, 0]	(-10°, 0%), (0°, 0%)	[85.3, 0.0]	85.3
10	[A ₁₀ , A ₂₈]	[2, 0]	(0°, 0%), (-10°, 0%)	[0.0, 16.35]	16.35
11	[A ₁₁ , A ₁₃]	[0, 1]	(0°, 0%), (-10°, -40%)	[0.0, 689.63]	689.63
12	[A ₁₃ , A ₁₄]	[1, 0]	(0°, 0%), (+10°, 0%)	[0.0, 45.32]	45.32
13	[A ₁₁ , A ₁₄]	[0, 3]	(0°, 0%), (+10°, 0%)	[0.0, 5.54]	5.54
14	[A ₁₂ , A ₁₃]	[0, 2]	(-10°, 0%), (0°, 0%)	[141.34, 0.0]	141.34

Table 5: The critical zones and conflicts resolution strategies under a scenario of airspace closing.

CZ	Aircraft IDs	Route segment	Decisions by aircraft	Cost for each aircraft	Total cost
1	[A ₁₈ , A ₂₂]	[0, 0]	(-20°, -40%), (0°, 0%)	[985.43, 0.0]	985.43
2	[A ₁₂ , A ₂₄]	[0, 0]	(-10°, 0%), (0°, 0%)	[104.29, 0.0]	104.29
3	[A ₃ , A ₆]	[0, 0]	(-20°, 0%), (0°, 0%)	[320.37, 0.0]	320.37
4	[A ₁₁ , A ₁₄]	[0, 0]	(-10°, 0%), (0°, 0%)	[811.61, 0.0]	811.61
5	[A ₁₈ , A ₂₂]	[1, 0]	(-10°, 0%), (0°, 0%)	[54.15, 0.0]	54.15

From these scenarios, the total cost of the system for the nominal situation, high airspace congestion, and closing the airspace are 2083.16 mi²/hr, 2463.32 mi²/hr, 2275.85 mi²/hr, respectively which are not significantly different, except for the cost of high airspace congestion. However, in the last scenario the passengers do not reach their desired destinations. Therefore, the cost of customer dissatisfaction is significantly high. Furthermore, as the number of the aircraft getting closer to the landing merging point increases, their maneuvers become more limited due to the high traffic congestion. Situations are exacerbated, especially in the emergency circumstances where the number of airplanes in the airspace increases or aircraft are forced to land on the nearest airport.

5. ANALYSIS

In this section, a sensitivity analysis is performed to show the variation cost from the original flight plans for different minimum safety distances and different flexibility levels in decision making procedures. The concept of “flexibility” in decision-making contexts refers to the number of available options. In this case, a more flexible approach would imply more options (for trajectory and velocity change) that can be selected by the pilots in a conflict situation.

Figure 5 Figure 6 Figure 7 illustrate the cost of deviation from the primary flight routes for various separation distances from 3 to 10 miles for two scenarios of decision flexibility. As such, the inflexible decision approach in the analysis considers three levels for the trajectory and velocity change (i.e., 9 possible individual strategies) as presented in the sets of $\{-20^\circ, 0, +20^\circ\}$ and $\{-40\%$,

0, 40% }, respectively. In the first scenario shown in Figure 5, the flexible decision approach considers five levels for both the trajectory and velocity change following the sets of $\{-20^\circ, -10^\circ, 0, +10^\circ, +20^\circ\}$ and $\{-40\%, -20\%, 0, 20\%, 40\%\}$ respectively.

According to Figure 5, the flexible decision approach inflicts less cost to the system compared to the inflexible approach however, it increases the computation time of the algorithm. In addition, as the minimum separation distance increases, a higher cost is imposed to the system.

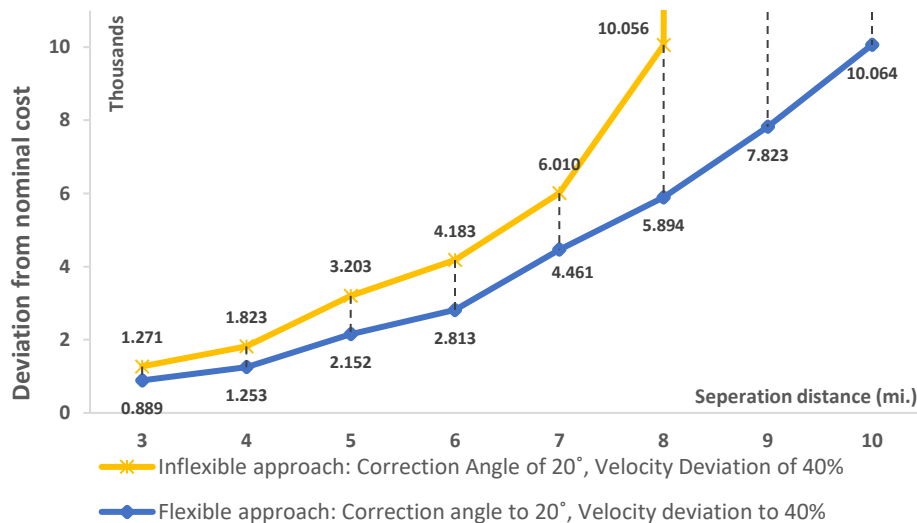


Figure 5: The cost of deviation from the original flight plans.

In Figure 6, available options for velocity change increases to five levels in the set of $\{-40\%, -20\%, 0, 20\%, 40\%\}$. On the other hand, the allowable trajectory changes for the flexible approach is 20° diagonally from both directions defined with three levels.

Figure 7 concerns more about the trajectory flexibility. As such in flexible approach, five levels are set for airplane trajectory changes in the set of $\{-20^\circ, -10^\circ, 0, +10^\circ, +20^\circ\}$ while the velocity changes have three levels in the set of $\{-20\%, 0, 20\%\}$.

In general, the flexible approach results in a lower cost compared to the inflexible approach. According to Figures 5 through 7, the cost of deviation from the original flight plans is lower while there is the flexibility in both trajectory and velocity in comparison to the flexibility in either trajectory or velocity. It can be concluded that as the number of available options for trajectory and velocity change increase, the cost imposed to the system reduces. As such, pilots do not necessarily take intense maneuvers in a conflict situation, and, subsequently, fuel consumption is reduced.

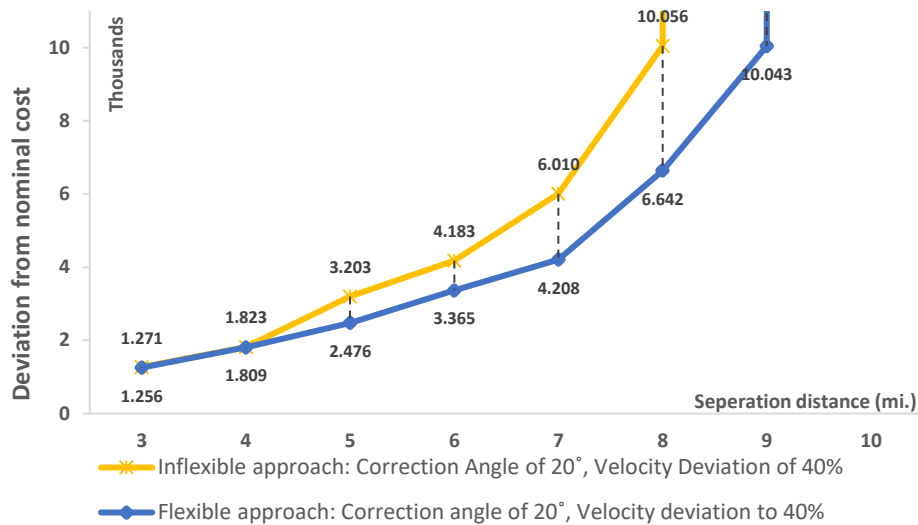


Figure 6: The cost of deviation from the original flight plans.

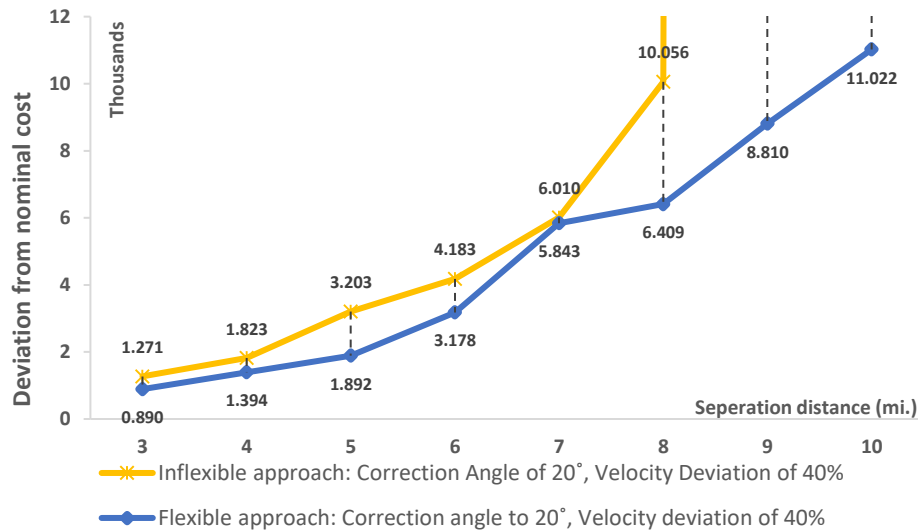


Figure 7: The cost of deviation from the original flight plans.

6. CONCLUSION

In summary, due to the growth of airspace congestion, the problem of conflict detection and resolution is becoming a challenging problem in air traffic management. In this study, we propose an algorithm based on game theory to resolve the problem of conflicts among multiple aircrafts moving in a shared airspace and provide a new set of maneuvers that deviate minimally from the original routes. The proposed game model predicts the pilot behavior at different stages of flight, such as mid-air and merging for landing scenarios. This approach will enhance the cognitive abilities of the pilot and avoid potential collisions thereby improving flexibility in handling variations to normal conditions. The primary focus of this study is to predict the occurrence of

conflicts among airplanes in the future and assist controllers in the conflict resolution by providing new movement scenarios for the aircrafts. This study generally improves decision making strategies in scenarios with humans involved. The efficiency of this model affects the efficiency of the air traffic controllers as they are the main components of the air traffic management.

In this study, the airspace is approximated to a 2-D grid and CZs are fixed by route change and speed change. For the future research, it would be beneficial to consider changes of altitude as another option for the conflict resolution to facilitate the application of the proposed game theory model in a more realistic situation. Additionally, the model could be further generalized by adding the cost of delay and customer dissatisfaction to the payoff function. Through making several experiments for different scenarios, it is realized that to have the better solutions with less cost, the scope of the problem related to each CZ should not be limited to only one route segment in the aircraft path. By considering more route segments, the radical changes in flight characteristics are reduced, which makes the flying path smoother. Therefore, it improves the operation of the model, especially in the landing merging point where high traffic congestion is expected. In addition, it is realized that high congestion in some parts of the airspace would decrease and limit the ability of the aircraft maneuvers with respect to the CZs. Thus, an additional model could be synchronized with the current one to balance the number of aircrafts in various parts of the airspace, by changing the aircraft flight characteristics to avoid unnecessary congestions.

For future work, to make the model more practical and realistic, we need to consider altitude as the 4th dimension of the airspace in the model. This can enable us to learn the implication of altitude change in resolving the conflict zones. Additionally, to improve the efficiency of the model, the decision horizon can be increased in order to reduce the total cost of deviation from the original path. In this case, the decision horizon is not limited only to a route segment, therefore, less drastic maneuvers are required to avoid and resolve conflicts in the airspace. Moreover, other features can be added to the payoff function to simulate the air traffic control interest effectively. For example, aircraft specification, variables representing time sensitivity, weather and airlines preferences would significantly improve the practicality of the model and align it with other Performance Based Navigation initiatives. Finally, as this study focuses on minimizing the deviation from the original path, a model is required to devise optimized flight plan considering weather situation and traffic congestion in each sector of the airspace.

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